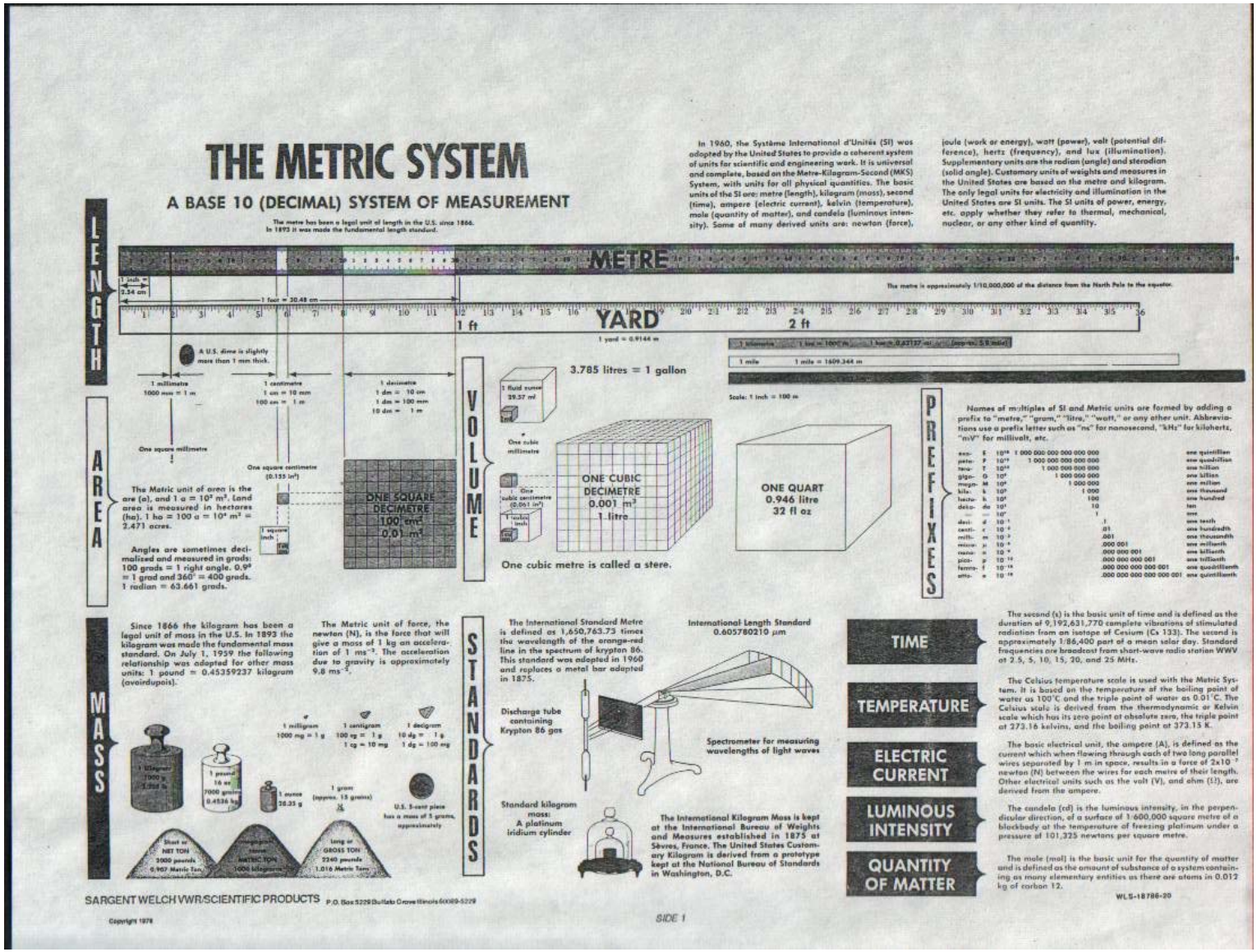
A103 MATH REFRSHER PACKET



Powers of Ten & Scientific Notation

1000000000000000000	10^{18}	exa	E	one quintillion
100000000000000000	10^{17}			one hundred quadrillion
10000000000000000	10^{16}			ten quadrillion
1000000000000000	10^{15}	peta	P	one quadrillion
100000000000000	10^{14}			one hundred trillion
10000000000000	10^{13}			ten trillion
1000000000000	10^{12}	tera	T	one trillion
100000000000	10^{11}			one hundred billion
10000000000	10^{10}			ten billion
1000000000	10^{9}	giga	G	one billion
100000000	10^{8}			one hundred million
10000000	10^{7}			ten million
1000000	10^{6}	mega	M	one million
100000	10^{5}			one hundred thousand
10000	10^{4}			ten thousand
1000	10^{3}	kilo	k	one thousand
100	10^{2}	hecto	h	one hundred
10	10^{1}	deca	da	ten
1	10^{0}			one
10^{-1}	.1	deci	d	one tenth
10^{-2}	.01	centi	c	one hundredth
10^{-3}	.001	milli	m	one thousandth
10^{-4}	.0001			one ten thousandth
10^{-5}	.00001			one hundred thousandth
10^{-6}	.000001	micro	m	one millionth
10^{-7}	.0000001			one ten millionth
10^{-8}	.00000001			one hundred millionth
10^{-9}	.000000001	nano	n	one billionth
10^{-10}	.0000000001			one ten billionth
10^{-11}	.00000000001			one hundred billionth
10^{-12}	.000000000001	pico	p	one trillionth
10^{-13}	.0000000000001			one ten trillionth
10^{-14}	.00000000000001			one hundred trillionth
10^{-15}	.000000000000001	femto	f	one quadrillionth
10^{-16}	.0000000000000001			one ten quadrillionth
10^{-17}	.00000000000000001			one hundred quadrillionth
10^{-18}	.000000000000000001	atto	a	one quintillionth

Scientific Notation — Here's how to do it:

- Move the decimal point of the original number so that it is to the right of the first [left] non-zero digit.
- Rewrite this first part of the number keeping in mind significant digits.
- Count how many places you moved the decimal point.
- Put this number as the exponent of 10 in the new multiplier.
- Rewrite the whole thing.

149,600,000,000	=	1.496×10^{11}
0.000000000000000000000021	=	2.1×10^{-24}
57,010	=	5.701×10^{4}
3,333,330	=	3.33333×10^{6}
0.567	=	5.56×10^{-1}
0.00034	=	3.4×10^{-4}

SCIENTIFIC NOTATION

When you use the word astronomical to describe something, it means BIG! And for good reason; things in astronomy are big. Take for instance the average distance between the Sun and the Earth - 149,600,000,000 meters. It is a fairly clumsy number to have to use in calculations. Instead, scientists have a short hand way of writing and using really big numbers, called [oddly enough] scientific notation. The Sun-Earth distance could then be written as 1.496×10^{11} meters. This is easier than writing out all those zeros [less chance for error].

The same principle works with really small numbers. These are decimal numbers [numbers that are less than 1 but greater than 0]. In this case the exponent of 10 will be a negative number, to indicate that the decimal point is moved in the other direction. Try this one - the density of gas in interstellar space is 0.000000000000000000000021 g/cm^3. Could you imagine having to work with that?

149,600,000,000	=	1.496×10^{11}
0.000000000000000000000021	=	2.1×10^{-24}

Here's how to do it:

	Example 1	Example 2
Recognize where the decimal point is in the number. The decimal point of any whole number is to the right of the number.	**149600000000.**	**0.000000000000000000000021**
Move the decimal point so that it is to the right of the first [left] non-zero digit	**1.49600000000**	**000000000000000000000021.**
Rewrite this first part to the number of significant digits you want. [See SIG FIGURES]	**1.496**	**2.1**
Count how many places you moved the decimal point.	**11 places to the left**	**24 places to the right**
Put this number as the exponent of 10 in the new multiplier.	**x 10^{11}**	**x 10^{-24}**
Rewrite the whole thing.	**1.496×10^{11}**	**2.1×10^{-24}**

MULTIPLYING OR DIVIDING BY 10

67 x 10	=	670
67 x 1000	=	67000
67 / 10	=	6.7
67 / 1000	=	0.067

SCIENTIFIC CALCULATORS

Hooray for scientific calculators! They certainly make life a whole lot easier. However, they can only calculate what you put in. And they can't evaluate the answers. That's what you do. The final question of any problem should be - does the answer make sense?

When using scientific calculators for problems involving scientific notation, there is a special key to use. It is either marked EXP or EE. What this key does is tell the calculator to multiply by 10 raised to some power. For example, let's input 6.0×10^{12} [the distance light travels in a year].

First, press 6.0 Then, press the EXP or EE key Then, press 12

You should see on the screen on ordinary scientific calculator

6.0 12

or

6.0e+012 on WINDOWS scientific calculator

Be aware of what this looks like, since your answers may pop out in this format. So, if you calculate 55,555,555 x 22,222,222 you're answer will look something like:

1.2345678 15

or

1.2345678e+015

This should be read as 1.2345678×10^{15}

UNITS

You must always make sure your units match - in other words, don't use miles and kilometers in the same problem.

CONVERT MISMATCHED UNITS FIRST!!!

LENGTH

1 m = 100 cm = 10^{-3} km = 1000 mm
= 1.0936 yd = 3.281 ft = 39.37 in
1 in = 2.54 cm
1 ft = 12 in = 30.48 cm
1 yd = 3 ft = 91.44 cm
1 mi = 1.6093 km
1 km = 0.6215 mi

WAVELENGTH

1 nm = 10^{-9} meter
1 Å = 10^{-10} m = 0.1 nm

SURFACE AREA

A = length × width (rectangle)
$= \pi \cdot r^2$ (circle)
$= 4\pi \cdot r^2$ (sphere)
units are squared units of length, *ie*- m^2 or cm^2

VOLUME

V = length × width × height (rectangular solid)
$= {}^4/_3 \cdot \pi \cdot r^3$ (sphere)
units are cubed units of length, *ie*- m^3, cm^3

ANGLES

1 degree = 60 arcminutes = 60' = 3600"
1 arcmin = 60 arcseconds = 60"
1 radian = 360°/2π = 57.30° = 206,265"

SPEED

speed = distance/time
1 km/hr = 0.2778 m/sec = 0.6215 mi/hr
1 mi/hr = 0.4470 m/sec = 1.609 km/hr

TIME

1 hr = 60 min = 3600 sec
1 day = 24 hr = 1440 min = 86,400 sec
1 year = 365.244 days = 8765.856 hr
= 525,951 min = 31,557,082 sec ≈ 3 x 10^7 sec

TEMPERATURE

K is Kelvin, C is degrees Celsius,
and F is degrees Fahrenheit
K = C + 273.15
F = 32 + 1.8 x C
C = $^5/_9$ (F - 32)

MASS

weight ≠ mass; weight = force
1 kg = 1000 g ≈ 2.205 lb (only at Earth's surface)
1 g = 10^{-3} kg = 1000 mg ≈ 0.03527 oz
1 lb ≈ 453.59 g (pound is weight - not mass)

DENSITY

density = mass/volume units, *ie*- kg/m^3

ENERGY

1 J = 0.2388 calories = 10^7 ergs
= 6.2415 × 10^{18} eV
1 erg = 1 g·cm^2/sec^2 = 10^{-7} J
1 kW·h = 3.6 MJ
1 eV = 1.602 × 10^{-19} J = 1.602 × 10^{-12} ergs

POWER

power = energy ÷ time
1 W = 10^7 erg/sec

SI stands for Système International; it is the metric system of units used internationally by most scientists.

THE BASIC SI UNITS ARE:

meters	m	length
seconds	sec	time
kilograms	kg	mass
kelvin	K	temperature

THE DERIVED SI UNITS ARE:

newton	N	force	N = kg·m/sec^2
joule	J	energy	J = N·m = kg·m^2/sec^2
watt	W	power	W = J/s = kg·m^2/sec^3

Å = Angstrom	kg = kilogram	mm = millimeter
AU = astronomical unit	km = kilometer	Mpc = megaparsec
C = coulomb(charge)	kW = kilowatt	nm = nanometer
cm = centimeter	lb = pound	oz = ounces
eV = electron volt	ly = light-year	pc = parsec
ft = feet	m = meter	sec = second
g = gram	mg = milligram	W = watt
hr = hour	mi = mile	yr = year
in = inch	min = minute	yd = yard
J = joule	MJ = megajoule	

CONSTANTS & ASTROPHYSICAL QUANTITIES

Velocity of light	c	$= 3.00 \times 10^{8}$ m/sec
Gravitational constant	G	$= 6.67 \times 10^{-11}$ N·m²/kg²
Planck constant	h	$= 6.626 \times 10^{-34}$ J·sec
Boltzmann constant	k	$= 1.38 \times 10^{-23}$ J/K
Rydberg constant	R	$= 1.097 \times 10^{7}$ /m
Stefan-Boltzmann constant	σ	$= 5.67 \times 10^{-8}$ W/m²·K⁻⁴
Mass of hydrogen atom (proton)	m_H	$= 1.673 \times 10^{-27}$ kg
Mass of helium atom	m_{He}	$= 6.645 \times 10^{-27}$ kg
Mass of electron	m_e	$= 9.109 \times 10^{-31}$ kg
Charge of electron	e	$= 1.602 \times 10^{-19}$ C
Electron volt	1 eV	$= 1.602 \times 10^{-19}$ J
Astronomical Unit (Sun-Earth distance)	AU	$= 1.496 \times 10^{11}$ m
Parsec	pc	$= 206{,}265$ AU
		$= 3.262$ ly
		$= 3.086 \times 10^{16}$ m
Light-year	ly	$= 6.324 \times 10^{4}$ AU
		$= 0.307$ pc
		$= 9.461 \times 10^{15}$ m
Mass of Earth	$M_\oplus$	$= 5.98 \times 10^{24}$ kg
Radius of Earth at equator	$R_\oplus$	$= 6378$ km
Orbital velocity of Earth	$V_\oplus$	$= 29.8$ km/sec
Mass of Sun	$M_\odot$	$= 1.99 \times 10^{30}$ kg
Radius of Sun	$R_\odot$	$= 6.96 \times 10^{5}$ km
Luminosity of Sun	$L_\odot$	$= 3.90 \times 10^{26}$ W
Effective temperature of Sun	T_{eff}	$= 5780$ K
Mass of Moon	$M_☽$	$= 7.35 \times 10^{22}$ kg
		$= 0.0123\ M_\oplus$
Radius of Moon	$R_☽$	$= 1738$ km
		$= 0.272\ R_\oplus$
Distance from Earth to Moon	$d_☽$	$= 3.84 \times 10^{5}$ km
Sidereal month	$P_☽$	$= 27.3$ days
Synodic month		$= 29.5$ days
Distance of Sun from center of Galaxy	$R_\odot$	$= 8.5$ kpc
Velocity of Sun about galactic center	$V_\odot$	$= 220$ km/sec
Diameter of Galaxy	D_∞	$= 120$ kpc
Mass of Galaxy	M_∞	$= 7 \times 10^{11}\ M_\odot$

Many of these constants are known to better accuracy, however, for the purposes of this course, the given accuracy will be sufficient.