Mathematics Review for Astronomy

This page provides a review of some of the mathematical concepts you will apply in the lab and lecture of this course.

Fractions and Percentages %

Fractions are a part of every-day life and they are also used in science. A fraction with the same numerator (number on top) as another but with a smaller denominator (number on bottom) is a *larger* number. For example 1/2 is *bigger* than 1/3 which is bigger than 1/4, etc. That is because if you divide 1 by 2, you get 0.5 but 1 divided by 3 is only 0.333 and 1 divided by 4 is only 0.25. The fraction 3/5 (=0.6) is bigger than 3/6 (=0.5) which is bigger than 3/7 (=0.4286).

Fractions are sometimes expressed as a **percent**. To express a fraction as a percentage, find the decimal form and multiply by 100. The percent symbol ``%'' simply means ``divide by 100.'' For example 1/2 = 0.5 = 0.5× 100% = 50%; 3/5 = 0.6 = 0.6× 100% = 60%. Some examples on converting percentages to fractions or decimals: 5.8% = 5.8/100 = 0.058; 0.02% = 0.02/100 = 0.0002; the Sun is 90% hydrogen means that 90 out of every 100 atoms in the Sun is hydrogen.

``Times'' and ``Factor of''

Several homework questions ask you to find some quantity and find out how many ``times'' smaller or larger it is than something else, e.g., star A is ____times larger than star B. This means star A = α × star B, and you must find the number α. Another example: A gallon is equivalent to 4 quarts. This means that 1 gallon is 4 **times** *bigger* than 1 quart since 1 gallon = 4 × 1 quart, or (1 gallon)/(1 quart) = 4/1. This also means that 1 quart is 4 **times** *smaller* than 1 gallon since 1 quart = 1/4 × (1 gallon), or (1 quart)/(1 gallon) = 1/4. Notice when ``times bigger'' is used and when ``times smaller'' is used.

The use of the phrase ``factor of'' is very similar to the use of ``times''. For example, 1 quart is a **factor of** 4 *smaller* than 1 gallon, or 1 gallon is a **factor of** 4 *bigger* than 1 quart.

Exponents

Warning

Your web browser needs to be able to format superscripts for the following sections! If the number ``$10^6$'' looks like ``106'', then your web browser cannot format superscripts and the following will be a bit confusing for that reason.
A shorthand way to express a quantity multiplied by itself one or more time is to use a superscript number called an **exponent**. So

$$a = a^1$$
$$a \times a = a^2 \text{ (not } 2 \times a\text{!)}$$
$$a \times a \times a = a^3 \text{ (not } 3 \times a\text{!)}$$
$$a \times a \times a \times a = a^4$$
$$a \times a \times a \times a \times a = a^5$$

The quantity ``*a* squared" means $a^2$, ``*a* cubed" means a^3, and more generally, ``*a* to the *n*th power" is a^n.

Some special rules apply when you divide or multiply numbers raised to some power.

When you have a^n multiplied by a^m, the result is *a* raised to a power that is the sum of the exponents:

$$a^n \times a^m = a^{n+m}$$

When you have a^n divided by a^m, the result is *a* raised to a power that is the difference of the exponents---the exponent on the bottom is subtracted from the exponent on the top:

$$(a^n) / (a^m) = a^{n-m} \text{ (not } a^{m-n}\text{!)}$$

When you have a^n raised to a power *m,* you multiply the exponents:

$$(a^n)^m = a^{n \times m} = a^{nm} \text{ (not } a^{n+m}\text{!)}$$

Negative exponents are used for reciprocals:

$$1 / a = a^{-1},\ 1 / (a^2) = a^{-2},\ 1 / (a^3) = a^{-3},\ 1 / (a^4) = a^{-4}, \text{ etc.}$$

Scientific calculators have a ``$y^x$" key or a ``x^y" that takes care of raising numbers to some exponent. Some fancy calculators have a ``^" key that does the same thing. Some calculators have ``x^2" and ``x^3" keys to take care of those frequent squaring or cubing of numbers. Check your calculator's manual or your instructor. The Basic Skills Computer Lab has some excellent software that can improve your skills with exponents. Try it out!

Roots

The **square root** of a quantity is a number that when multiplied by itself, the product is the original quantity:

$$Sqrt[a] \times Sqrt[a] = \text{a.}$$

Some examples: *Sqrt*[1] = 1 because 1 × 1 = 1; *Sqrt*[4] = 2 because 2 × 2 = 4; *Sqrt*[38.44] = 6.2 because 6.2 × 6.2 = 38.44; *Sqrt*[$25A^2$] = 5A because 5A × 5A = $25A^2$.

A square root of a number less than 1, gives a number *larger* than the number itself: *Sqrt*[0.01] = 0.1 because 0.1 × 0.1 = 0.01 and *Sqrt*[.36] = 0.6 because 0.6 × 0.6 = 0.36.

The **cube root** of a quantity is a number that when multiplied by itself two times, the product is the original quantity:

$$Cube\text{-}Root[a] \times Cube\text{-}Root[a] \times Cube\text{-}Root[a] = \text{a.}$$

Scientific calculators have `` " and sometimes ``$\sqrt[3]{x}$" keys to take care of the common square roots or cube roots. An expression $\sqrt[n]{a}$ means the *n*th root of *a.* How can you use your calculator for something like that? You use the fact that the *n*th root of *a* is *a* raised to a fractional exponent of *1/n.* So we have:

$$\sqrt[n]{a} = a^{1/n}$$

$$Sqrt[a] = \sqrt[2]{a} = a^{1/2}$$

$$\textit{Cube-root}[a] = \sqrt[3]{a} = a^{1/3}$$

When you raise some *n*th root to some power *m,* you simply multiply the exponents as you did above for $(a^n)^m$:

$$(\sqrt[n]{a})^m = (a^{1/n})^m = a^{1/n \times m} = a^{m/n}.$$

So $(a^6)^{1/2} = a^{6\times (1/2)} = a^3$ and $(a^{1/2})^6 = a^{(1/2)\times 6} = a^3$. But if you have $a^{1/n}$ multiplied by a^m, you add the exponents since you are *not* raising $a^{1/n}$ to some power *m*: $a^{1/2} \times a^6 = a^{(1/2)+6} = a^{6\ \&\ 1/2} = a^{13/2}$.

Powers of Ten

For numbers larger than 10, the power of 10 is a positive value and negative for numbers less than 1. For numbers between 0 and 10, the power is a positive fraction. In the examples that follow, notice what happens to the decimal point:

$10^0 = 1.$ = 1. with the decimal point moved 0 places

$10^1 = 10.$ = 1. with the decimal point moved 1 place to the right

$10^2 = 100.$ = 1. with the decimal point moved 2 places to the right

$10^6 = 1000000.$ = 1. with the decimal point moved 6 places to the right

and

$10^{-1} = 0.1$ = 1. with the decimal point moved 1 place to the *left*

$10^{-2} = 0.01$ = 1. with the decimal point moved 2 places to the *left*

$10^{-6} = 0.000001$ = 1. with the decimal point moved 6 places to the *left.*

The exponent of 10 tells you how many places to move the decimal point to the right for positive exponents or left for negative exponents. These rules come in especially handy for writing very large or very small numbers.

Scientific Notation

Since you will be working with very large and very small numbers, use scientific notation to cut down on all of the zeroes you need to write. Proper scientific notation specifies a value as a number between 1 and 10 (called the **mantissa** below) multiplied by some power of ten, as in

$$\textit{mantissa} \times 10^{\textit{exponent}}.$$

The power of ten tells you which way to move the decimal point and by how many places. As a quick review:
$10 = 1 \times 10^1$, $253 = 2.53 \times 100 = 2.53 \times 10^2$ and $15{,}000{,}000{,}000 = 1.5 \times 10^{10}$ which you will sometimes see written as 15×10^9 even though this is not proper scientific notation. For small numbers we have: $\frac{1}{10} = 1 \times 10^{-1}$, $\frac{1}{253} = \frac{1}{2.53 \times 100} = \frac{1}{2.53} \times 10^{-2}$ or about $0.395 \times 10^{-2} = 3.95 \times 10^{-3}$.

Multiplying and Dividing with Scientific Notation

When you multiply two values given in scientific notation, multiply the mantissa numbers and *add* the exponents in the power of ten. Then adjust the mantissa and exponent so that the mantissa is between 1 and 10 with the appropriate exponent in the power of ten. For example: $(3\times10^{10}) \times (6\times 10^{23})$, you'd have $3\times6 \times 10^{(10+23)} = 18. \times 10^{33} = 1.8\times10^{34}$.
When you divide two values given in scientific notation, divide the mantissa numbers and *subtract* the exponents in the power of ten. Then adjust the mantissa and exponent so that the mantissa is between 1 and 10 with the appropriate exponent in the power of ten. For example: $\frac{3\times10^{10}}{6\times10^{23}} = \frac{3}{6}\times 10^{10-23} = 0.5 \times 10^{-13} = 5 \times 10^{-14}$.
Notice what happened to the decimal point and exponent in the examples. You *subtract* one from the exponent for every space you move the decimal to the *right.* You *add* one to the exponent for every space you move the decimal to the *left.*

Entering scientific notation on your scientific calculator

Please read this if you have not used a scientific calculator for a while!
Most scientific calculators work with scientific notation. Your calculator will have either an ``EE" key or an ``EXP" key. That is for entering scientific notation. To enter 253 (2.53×10^2), you would punch *2* ⟶. ⟶*5* ⟶*3* ⟶**EE** or **EXP** ⟶*2.* To enter 3.95×10^{-3}, you would punch *3* ⟶. ⟶*9* ⟶*5* ⟶**EE** or **EXP** ⟶*3* ⟶[± key]. Note that if the calculator displays ``3.53 -14" (a space between the 3.53 and -14), it means 3.53×10^{-14} **NOT** 3.53^{-14}! The value of $3.53^{-14} = 0.00000002144 = 2.144\times10^{-8}$ which is vastly different than the number 3.53×10^{-14}. Also if you have the number 4×10^3 and you enter *4* ⟶× ⟶*1* ⟶*0* ⟶ **EE** or **EXP** ⟶*3,* the calculator will interpret that as $4 \times 10 \times 10^3 = 4 \times 10^4$ or ten times greater than the number you really want!
One other word of warning: the EE or EXP key is used only for scientific notation and NOT for raising some number to a power. To raise a number to some exponent use the ``$y^x$" or ``x^y" key depending on the calculator. For example, to raise 3 to the 4th power as in 3^4 enter 3 ⟶y^x or x^y ⟶4. If you instead entered it using the EE or EXP key as in 3 ⟶ EE or EXP ⟶4, the calculator would interpret that as 3×10^4 which is much different than $3^4 = 81$.

Metric System

The metric system is a much more logical and straightforward system than the old english system still used in the United States. Every other nation in the world uses the metric system, so in today's global economy, it is very advantageous to learn how to use the system. It is also the system used in science.

Prefixes

The metric system is based on the number 10, for example there are 10 millimeters in one centimeter, 1000 grams in one kilogram, 100 centimeters in one meter, 100 degrees between the freezing and boiling point of water, etc. Contrast this with the english system that has 12 inches to 1 foot, 16 ounces to 1 pound, 5,280 feet to one mile, 180 degrees Fahrenheit between the freezing and boiling point of water, ... egad! All of the units in the metric system use a common set of prefixes:

prefix	meaning	example
pico	$10^{-12} = 1/10^{12}$	1 picosecond = 10^{-12} second
nano	$10^{-9} = 1/10^{9}$	1 nanometer = 10^{-9} meter
micro	$10^{-6} = 1/10^{6}$	1 microgram = 10^{-6} gram
milli	$10^{-3} = 1/1000$	1 millikelvin = 10^{-3} kelvin
centi	$10^{-2} = 1/100$	1 centimeter = 1/100th meter
kilo	1000	1 kilogram = 1000 grams
mega	10^{6}	1 megasecond = 10^{6} seconds
giga	10^{9}	1 gigakelvin = 10^{9} kelvins
tera	10^{12}	1 terameter = 10^{12} meters

Length

The base unit of length in the metric system is the meter. It is a little over 1 yard long, more precisely 39.37 inches long. Here are some other conversions:

1 meter (m)	= 39.37 inches	= 1.094 yards (about one big step)
1 kilometer (km)	= 1000 meters	0.62137 mile
1 centimeter (cm)	= 1/100th meter	0.3937 inch (1/2.54 inch, about width of pinky))

Mass

The base unit of mass is the gram. Mass is the amount of material in something and is different than weight. Review the definitions of mass, weight and the difference between the two. There are 28.3495 grams in one ounce. A paperclip has a mass of roughly one gram. Here are some other conversions:

1 gram (g)	= 0.0353 ounce	produces 0.0022046 pounds **of weight** on the Earth
1 kilogram (kg)	= 1000 grams	produces 2.205 pounds **of weight** on the Earth
1 metric ton	= 1000 kilograms	produces 2,205 pounds **of weight** on the Earth

Temperature

The base unit of temperature is the kelvin, though the celsius is also used. The kelvin scale is measured from absolute zero---the temperature at which all motion stops, the absolute coldest temperature possible. The celsius scale is measured from the freezing point of pure water at sea level, but the intervals of temperature are the same as the kelvin scale. Water freezes at 0° celsius which is the same as 273 kelvin. Water boils at 100° celsius which is the same as 373 kelvin. The Fahrenheit scale in common usage in the United States has the freezing point of pure water at 32° fahrenheit and the boiling point at 212° fahrenheit at sea level. Below are conversion formulae for fahrenheit to the metric scales and vice versa. Remember to calculate the values in parentheses first!

To convert fahrenheit to celsius use: $° C = (5/9)(° F - 32)$.

To convert fahrenheit to kelvin use: $K = (5/9)(° F - 32) + 273$.

To convert celsius to fahrenheit use: $° F = (9/5)° C + 32$.

To convert kelvin to fahrenheit use: $° F = (9/5)(K - 273) + 32$.